EXERCISE SHEET FOR "THE DYNKIN CLASSIFICATION"

MARKUS REINEKE

Exercise 1: Consider the quiver K_m $1 \overset{(m)}{\Rightarrow} 2$ (with m parallel arrows). Show that the representations of K_m with $V_1 = \mathbb{C}$ are classified up to equivalence by the reduced row echelon form of matrices.

Exercise 2: Consider the quiver L_m with a single vertex i and two loops $\alpha, \beta : i \to i$. Show that the classification of representations of L_m on $V_i = \mathbb{C}^2$ depends on five independent continuous parameters. More precisely, show that there are mutually non-equivalent representations for which

$$(\mathrm{tr}(f_\alpha), \mathrm{tr}(f_\beta), \det(f_\alpha), \det(f_\beta), \mathrm{tr}(f_\alpha f_\beta))$$

assume arbitrary values.

Exercise 3: Describe the path algebras of the quivers L_m, K_m and $\bullet \to \bullet \leftarrow \bullet$.

Exercise 4: Work out some of the details and necessary verifications for the equivalence of categories between quiver representations and modules over the path algebra (well-definedness, compatibility with morphisms).

Exercise 5: Determine the positive roots of a D_4 diagram by iterative completion of squares for the quadratic form

$$d_1^2 + d_2^2 + d_3^2 + d_4^2 - d_1 d_4 - d_2 d_4 - d_3 d_4.$$

Exercise 6: Verify the following property of the Zariski topology on an affine space $\mathbb{C}^n$: if $\mathbb{C}^n$ is written as a disjoint union of finitely many locally closed subsets S_i, then precisely one of them is a dense subset.

Exercise 7: Describe the orbit of the representation

$$\mathbb{C}^2 \overset{\mathrm{id}}{\to} \mathbb{C}^2 \overset{\mathrm{id}}{\leftarrow} \mathbb{C}^2$$

by explicit polynomial inequalities (Hint: describe the orbit by rank conditions).

Exercise 8: Work out some of the details and necessary verifications for the equivalence of categories given by reflection functors (well-definedness, compatibility with morphisms, isomorphism of $S_i^+ S_i^-$ to the identity).

Exercise 9: Find a sequence $(i_1, \ldots, i_6)$ in $\{1, 2, 3\}$ such that, for the quiver Q given by

$$1 \to 2 \to 3,$$

i_k is a sink in $s_{i_{k-1}} \dots s_{i_1} Q$ for all $k = 1, \dots, 6$, and such that the

$$S^-_{i_1} \dots S^+_{i_{k-1}} E_{i_k}$$

list all six positive roots of Q.

Exercise 10: Construct an indecomposable representation corresponding to the highest root of E_6 by applying a sequence of five reflection functors to the indecomposable of dimension vector $(1, 1, 1, 1, 1, 1)$.