combinatorial
synergies

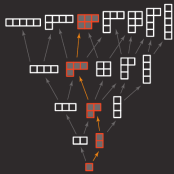Orientation Session, April 17, 2023

Program committee:

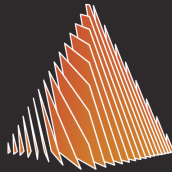Martina Juhnke, Thomas Kahle, Raman Sanyal, Christian Stump, Bernd Sturmfels

Today:

14:00 - 14:45 Introduction to the **9 themes** of the program

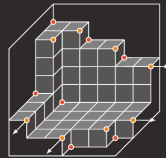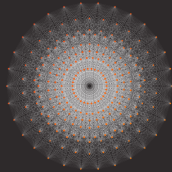14:50 - 15:10 How an SPP works (Carsten Balleier, DFG)

15:10 - 15:30 Q&A

ENUMERATION

STATISTICS

COMMUTATIVE ALGEBRA

MATHEMATICAL PHYSICS

DYNKIN CLASSIFICATION

CONVEXITY
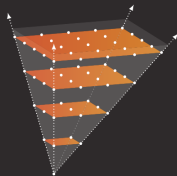
LATTICE POINTS
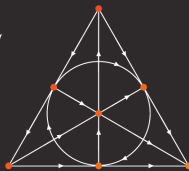
MATROIDS

NONLINEAR OPTIMIZATION

Theme T1: Enumeration

at the heart of combinatorics

as fingerprints of theories

We enumerate...

...dimensions of spaces   ...points on curves and in bodies   ...graphs with prescribed properties   ...classification types   ...irreducible characters   ...clusters for cluster algebras ...and many other structures

Theme T1: Enumeration

Combinatorial data

Building theories based on studying combinatorial data is like deriving physical laws from observations of phenomena in nature.
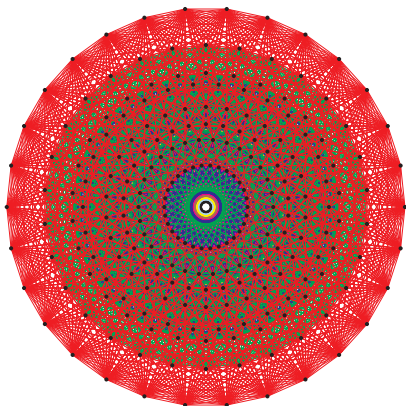
Some Synergies

- Counting is crucial in many classifications (▶ T2 Dynkin classification)

- f, g, h-vectors, positivity, reciprocity (▶ T2, ▶ T4 Matroids)

- face and intersection posets, order polytopes, associahedra, point configurations (▶ T4, ▶ T5 Convexity)

- Ehrhart theory (▶ T6 Lattice points)

- Amplituhedra stratifications, positive geometries (▶ T9 Physics)

Sample Question

- *Is it possible to semi-automatically find structural coincidences from numerical coincidences?*

Theme T2: Dynkin classification

fundamental symmetries

At the core of many classifications such as

- Coxeter groups
- Cluster algebras
- Frobenius manifolds

- quiver representations
- *elementary catastrophes*
- Calogero-Moser systems

# Theme T2: Dynkin classification

Some Synergies

- Weyl groups, Coxeter complexes (▶ T1, ▶ T3 Commutative algebra)
- reflection arrangements, permutahedra, associahedra
  (▶ T4 Matroids, ▶ T5 Convexity)
- E_8 lattice (▶ T6 Lattice points)
- Optimization under symmetries (▶ T8 Nonlinear Optimization)

Sample Question

- *Which data driven technologies are available to approach uniform theories?*

Theme T3: Commutative Algebra

bridge between

Algebra and Combinatorics
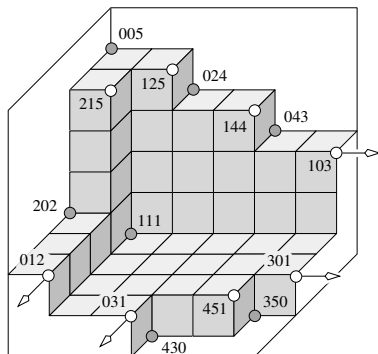
(monomial) ideals	⟿	simplicial complexes and posets
free resolutions of monomial ideals	⟿	staircase diagrams
algebraic invariants	⟿	combinatorial invariants
Hilbert function	⟿	face numbers
Betti numbers	⟿	simplicial homology

Theme T3: Commutative Algebra
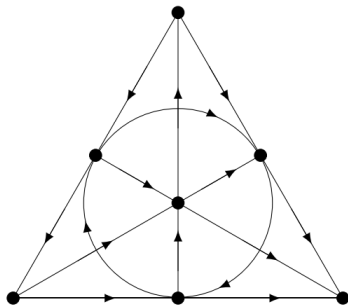
Some Synergies

- high-dimensional commutative algebra with symmetries: asymptotic behavior of invariants (▶ T1)

- Chow ring, Bergman fan (▶ T4 Matroids, ▶ T5 Convexity)

- Gröbner bases, toric rings (▶ T5, ▶ T6 Lattice points)

- binomial ideals, Markov bases (▶ T7 Statistics)

Some Questions

- *Asymptotic properties of high-dimensional algebraic/combinatorial objects?*

- *Extremal properties of Stanley-Reisner rings of Coxeter complexes?*

Theme T4: Matroids

ubiquitous in many areas

- Axiomatize linear independence
- Stanley-Reisner theory
- Hyperplane arrangements
- Positivity
- Hodge theory and Lorentzian polynomials

# Theme T4: Matroids

Some Synergies

- matroids as simplicial complexes, toric ideal (▶ T1, T3)
- matroid polytopes (▶ T6)
- gaussoids and semigraphoids (▶ T7)
- non-negative circuit polynomials (▶ T8)

Some Questions

- *Connections between classes of matroids and algebraic geometry?*
- *Realization spaces of valued oriented matroids?*

Theme T5: Convexity

gateway between

mathematical disciplines
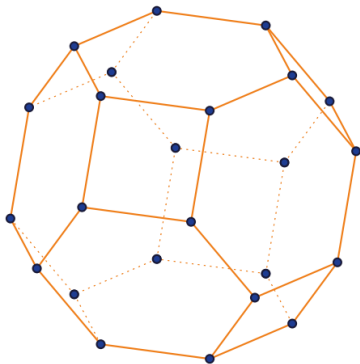
The permutahedron:

- geometry: diagonals of hermitian matrices with fixed spectrum
- faces: ordered set partitions counted Stirling numbers
- volume: spanning trees of complete graphs
- deformations: submodular functions

Theme T5: Convexity

Some synergies

- computational enumeration of faces, subdivisions, deformations (▶ T1)
- toric dictionary, permutahedral varieties, Kähler packages
  (▶T3 commutative algebra, ▶T6 lattice points)
- moment polytopes, Newton–Okounkov bodies, tropical varieties
- spectrahedra, amplituhedra, positive geometries
  (▶T8 nonlinear optimization, ▶T9 math. physics)

Some questions

- *What is a combinatorial theory of non-linear convex bodies?*
- *How to effectively test properties of moduli polytopes?*

Theme T6: Lattice points

Lattice points connect

the **discrete** and the **continuous**
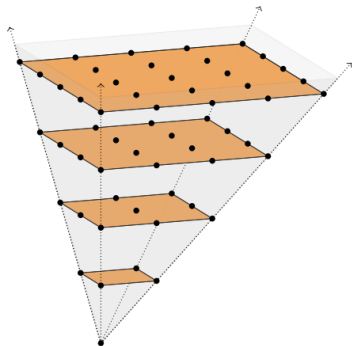
- lattices as translational symmetries of crystalline structures
- lattice points as anchors in packing problems (physics, coding theory)
- sets of lattice points as discrete structures (monomials, weights)

Theme T6: Lattice points

Some synergies

- lattice point enumerators as discrete volumes (▶ T1)
- Hilbert and Poincaré series of algebraic objects (▶ T3)
- computational methods for enumeration/classification (polymake, normaliz, ...)
- databases of Fano polytopes in combinatorial mirror symmetry (▶ T9)
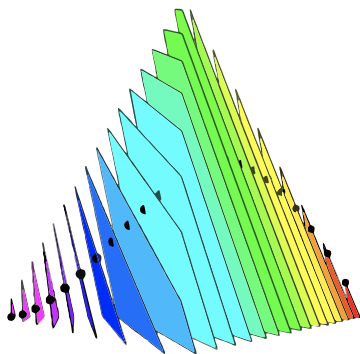
Some Questions

- *How to enumerate lattice points in nonlinear objects?*
- *How to effectively connect databases with a view towards applications?*

Theme T7: Statistics

Statistics and Combinatorics
have always been connected

From gambling to the ubiquitous
graphical calculus for correlation and
causation in science.

For example, likelihood geometry connects graphical models to real algebra and lattice polytopes. Voronoi cells for likelihood based distances reveal the geometry of parameter estimation.

Theme T7: Statistics

Some Synergies

- T1   Permutation models
- T2   Lagrangian Grassmannian
- T3   Markov bases
- T4   Marginal independence
- T5   Log-Voronoi cells
- T6   Lattice models
- T8   Wasserstein distances in statistics
- T9   Likelihood of scattering amplitudes

Some Questions

- *Which techniques allow the classification of 5-gaussoids?*
- *How does entropy reflect in convex geometry and algebraic geometry?*

Theme T8: Nonlinear Optimization

Optimizing a polynomial functions under polynomial constraints connects combinatorics, algebra and geometry.

Relaxing this NP-hard problem yields problems about spectrahedra and PSD-matrices.
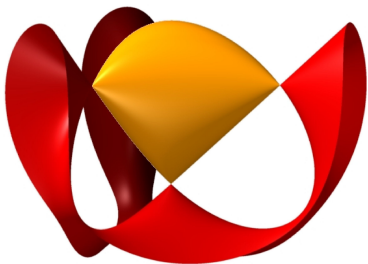
For example,

- SONCs provide matroid based nonnegativity certificates.
- Exploiting symmetry is essential.
- Boundaries of spectrahedra are often combinatorial.

Theme T8: Nonlinear Optimization

Some Synergies

- T1 Lorentzian polynomials
- T2 Optimization under symmetry
- T3 Gap vectors
- T4 SONCs
- T5 Gram spectrahedra
- T6 Nonlinear integer programming
- T7 Wasserstein distances in statistics
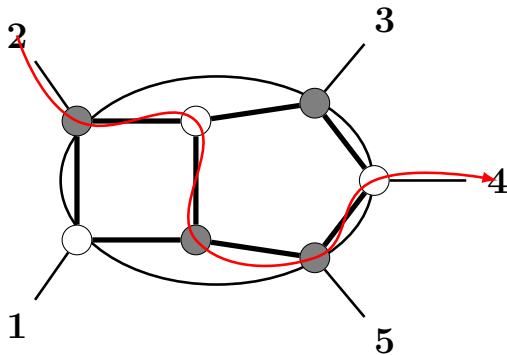- T9 Quantum correlations

Some Questions

- *What are combinatorial models for nonnegativity of polynomials?*
- *What is the combinatorics of spectrahedra from quantum information?*

We focus on *scattering amplitudes* in *particle physics*.

A key object is the **amplituhedron**. This is a projection of the **positive Grassmannian**. Its boundaries are given by **positroid cells**.

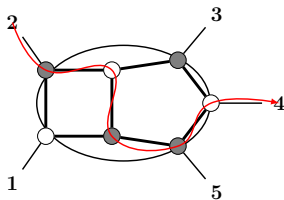

Description via **plabic graphs**.   Think: *Feynman diagrams*

Friends

Nima Arkani-Hamed, Johannes Henn, Thomas Lam, Lauren Williams, ...

Some Synergies

- T1   Triangulations of amplituhedra
- T2   Cluster algebras
- T3   Schubert calculus
- T4   Positroids
- T5   Cosmological polytopes
- T6   Lattice quantizers
- T7   Maximum likelihood degree
- T8   Quantum correlations

Some Questions

- *How to best write the canonical form of an amplituhedron?*
- *Do all positive geometries behave like convex polytopes?*

What Else   ... did we write about?

Combinatorics as cross section	→ math,cs,physics
International friends	→ strong support
Mathematical data	→ OEIS, FindStat
Research data management	→ MaRDI
Career development	→ young profs
Diversity	→ excellence
Network	→ coherence
Workshops	→ Magdeburg June
Conferences	→ FPSAC '24
Lectures	→ J Huh(Chow)
Guests	→ Mercator
REU	→ *summer of combinatorics*
Schools	→ PhD students, postdocs

best of all: **Your Projects**

The priority program

The DFG senate approved this priority program for 3+3 years.

First Funding period:
- **7.1 M €** central funds for network activities & ∼30 individual projects

Timeline:

May 2023	DFG call for proposals in the priority program
June 28-29 2023	24 Hours of Combinatorial Synergies (Magdeburg)
Sep 2023	Deadline for proposal submissions to DFG
Spring 2024	Decisions by DFG and start of the projects
starting early 2024	Requests for various network activities

www.combinatorial-synergies.de